Programmation 1

TD n°6

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$$\frac{}{\rho \vdash x := e \Rightarrow \rho[x \mapsto [\![e]\!]\rho]}\ (:=) \qquad \frac{}{\rho \vdash \texttt{skip} \Rightarrow \rho}\ (\texttt{Skip})$$

$$\frac{\rho \vdash c_1 \Rightarrow \rho' \quad \rho' \vdash c_2 \Rightarrow \rho''}{\rho \vdash c_1; c_2 \Rightarrow \rho''}\ (\texttt{Seq})$$

$$\frac{\rho \vdash c_1 \Rightarrow \rho'}{\rho \vdash \texttt{if}\ e\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \Rightarrow \rho'}\ (\texttt{if}_1)\ \text{si } [\![e]\!]\rho \neq 0 \qquad \frac{\rho \vdash c_2 \Rightarrow \rho''}{\rho \vdash \texttt{if}\ e\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \Rightarrow \rho'}\ (\texttt{if}_2)\ \text{si } [\![e]\!]\rho = 0$$

$$\frac{\rho \vdash c \Rightarrow \rho' \quad \rho' \vdash \texttt{while}\ e\ \texttt{do}\ c \Rightarrow \rho''}{\rho \vdash \texttt{while}\ e\ \texttt{do}\ c \Rightarrow \rho''}\ (\texttt{while})\ \text{si } [\![e]\!]\rho \neq 0 \qquad \frac{}{\rho \vdash \texttt{while}\ e\ \texttt{do}\ c \Rightarrow \rho}\ (\texttt{while}_{\text{fin}})\ \text{si } [\![e]\!]\rho = 0$$

FIGURE 1 – La sémantique opérationnelle à grands pas de IMP.

Exercise 1 :
Let $\texttt{while}\ 1\ \texttt{do skip}$. By considering the form of derivations, explain why, for any environment ρ, there is no environment ρ' such that $\rho \vdash \texttt{while}\ 1\ \texttt{do skip} \Rightarrow \rho'$.

Exercise 2 :
Given two programs c_1, c_2, we say that c_1 and c_2 are equivalent, denoted as $c_1 \sim c_2$ iff for all environments ρ, ρ', $(\rho \vdash c_1 \Rightarrow \rho') \Leftrightarrow (\rho \vdash c_2 \Rightarrow \rho')$.

1. Show that $\texttt{while}\ e\ \texttt{do}\ c \sim \texttt{if}\ e\ \texttt{then}\ (c;\ \texttt{while}\ e\ \texttt{do}\ c)\ \texttt{else skip}$.
2. What are the programs equivalent to $\texttt{while}\ 1\ \texttt{do skip}$?

Exercise 3 :
In the course, you saw a version of the IMP language without Boolean expressions. In your compiler, you had defined these expressions as :

$$b ::= \texttt{True} \mid \texttt{False} \mid e \dot{=} e \mid e \dot{<} e \mid \dot{\neg} b \mid b \dot{\vee} b \mid b \dot{\wedge} b$$

1. Give an operational semantics for Boolean expressions.
2. Modify your rules so that they implement the "or" such that : when evaluating $b_1 \dot{\vee} b_2$, b_2 is not evaluated if b_1 evaluates to True. Same thing for "and".
3. Modify your rules so that they implement the parallel evaluation of the "or". Same thing for the "and".
4. In the three deduction systems obtained, what can be said about the number of derivations of a triple $(b, \rho) \rightarrow \_$?

Exercise 4 :

One of the objectives of the following courses will be to introduce the mathematical tools necessary to be able to define a denotational semantics of IMP : $[\![c]\!]\rho = \rho'$, which means that the execution of the program c in the environment ρ leads to the environment ρ'. This «functional» writing suggests that the environment ρ' is fully determined by c and ρ.

1. Show this property for the operational semantics : for all c, ρ, ρ_1, ρ_2, if $\rho \vdash c \Rightarrow \rho_1$ and $\rho \vdash c \Rightarrow \rho_2$ then $\rho_1 = \rho_2$.
2. We consider the non-deterministic language given by the following syntax :

$$c ::= \texttt{skip} \mid x := e \mid c; c \mid \texttt{if } e \texttt{ then } c \texttt{ else } c \mid \texttt{while } e \texttt{ do } c \mid c \vee c$$

   where the instruction $c_1 \vee c_2$ signifies «execute c_1 or execute c_2, non-deterministically». Propose operational semantics for this language.