Tree Automata and their Applications

TD n°2 : Decision problems & tree homomorphisms

2021-2022

Exercise 1 : About Nutts

A bottom-up tree transducer (NUTT) is a tuple $U = (Q, \mathcal{F}, \mathcal{F}', Q_f, \Delta)$ where Q is a finite set (of states), $\mathcal{F}$ and $\mathcal{F}'$ are finite ranked sets (of input and output), $Q_f \subseteq Q$ (final states) and Δ is a finite set of rules of the form :

- $f(q_1(x_1), ..., q_n(x_n)) \to q(u)$ where $f \in \mathcal{F}$ and $u \in T(\mathcal{F}', \{x_1, ..., x_n\})$
- $q(x_1) \to q'(u)$ where $u \in T(\mathcal{F}', \{x_1\})$.

We say that U is linear when the right side of the rules of Δ are. This defines a rewrite system $\to_U$ on $T(\mathcal{F} \cup \mathcal{F}' \cup Q)$. The relation induced by U is then $\mathcal{R}(U) = \big\{(t, t') \mid t \in T(\mathcal{F}),\, t' \in T(\mathcal{F}'),\, t \to_U^* q(t'),\, q \in Q_f\big\}$.

1) Prove that tree morphisms are a special case of NUTT that is if $\mu : T(\mathcal{F}) \longrightarrow T(\mathcal{F}')$ is a morphism, then there exists a NUTT U_μ such that $\mathcal{R}(U_\mu) = \big\{(t, \mu(t)) \mid t \in T(\mathcal{F})\big\}$. Be sure that if μ is linear then U_μ is too.
2) Prove that the domain of a NUTT U, that is $\big\{t \in T(\mathcal{F}) \mid \exists t' \in T(\mathcal{F}'),\, (t, t') \in U\big\}$, is recognizable.
3) Prove that the image of a recognizable tree language L by a linear NUTT U, that is $\big\{t' \in T(\mathcal{F}') \mid \exists t \in L,\, (t, t') \in U\big\}$, is recognizable.

Exercise 2 : Decision problems

We consider the **(GII)** problem (ground instance intersection) :

Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and $\mathcal{A}$ a NFTA

Question : Is there at least one ground instance of t accepted by $\mathcal{A}$?

1) Suppose that t is linear. Prove that **(GII)** is P-complete.
2) Suppose that $\mathcal{A}$ is deterministic. Prove that **(GII)** is NP-complete.
3) Prove that **(GII)** is EXPTIME-complete.
   hint : for the hardness, reduce the intersection non-emptiness problem (admitted to be EXPTIME-complete).
4) Deduce that the complement problem :
   Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and linear terms t_1, ..., t_n
   Question : Is there a ground instance of t which is not an instance of any t_i ?
   is decidable.

Exercise 3 : Path closures

Let us revisit the example from last week : $\mathcal{F} = \big\{f(2), g(1), a(0)\big\}$. Consider the set $M(t)$ of terms which have a ground instance of the term $t = f\big(a, g(x)\big)$ as a subterm, i.e. $M(t) = \Big\{C\big[f\big(a, g(u)\big)\big] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F})\Big\}$.

Prove that $M(t)$ is not recognizable by a finite union of languages recognizable by a top-down DFTA.

Hint : You can use without proof the following fact (prove it if you have time) : let t be a tree. The path language $\pi(t)$ is defined by :

- *if t is a constant,* $\pi(t) = \{t\}$
- *if* $t = f(t_1, ..., t_n)$, $\pi(t) = \cup_{i=1}^{n} \{fiw \mid w \in \pi(t_i)\}$

Let L be a tree language. The path language of L is $\pi(L) = \cup_{t \in L} \pi(t)$. *The path closure of L is defined by*

$$pathclosure(L) = \{t \mid \pi(t) \subseteq \pi(L)\}$$

L is recognizable by a top-down DFTA iff L is recognizable and path closed, i.e. $L = pathclosure(L)$.

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Homework for next week : Direct images of an homomorphism

Let $\mathcal{F} = \{f/2, g/1, a\}$ and $\mathcal{F}' = \{f'/2, g/1, a\}$. Let us consider the tree homomorphism h determined by h_F defined by : $h_{\mathcal{F}}(f) = f'(x_1, x_2), h_{\mathcal{F}}(g) = f'(x_1, x_1)$, and $h_{\mathcal{F}}(a) = a$.

1. Is $h(\mathcal{T}(\mathcal{F}))$ recognizable ? Explain.
2. Let $L_1 = \{g^i(a) | i \geq 0\}$, then L_1 is a recognizable tree language, is $h(L_1)$ recognizable ? Explain.

Note : You can send the homework by mail to asuresh@lsv.fr, or hand it to me in person next time that we reconvene for the TD.