Tree Automata and their Applications

TD n°1 : Recognizable Tree Languages and Finite Tree Automata

2021-2022

Exercise 1 : First constructions of Tree Automatas

Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down DFTA for the set $G(t)$ of ground instances of the term $t = f(f(a,x), g(y))$ which is defined by :

$$G(t) = \Big\{ f\big(f(a,u), g(v)\big) \mid u, v \in T(\mathcal{F}) \Big\}$$

Exercise 2 : What is recognizable by an FTA ?

Are the following tree languages recognizable (by a bottom-up FTA) ?

- $\mathcal{F} = \{g(1), a(0)\}$ and L the set of ground terms of even height.
- $\mathcal{F} = \{f(2), g(1), a(0)\}$ and L the set of ground terms of even height.

Exercise 3 : Bottom-up vs Top-down

1) Recall why bottom-up NFTAs, bottom-up DTAs and top-down NFTAs have the same expressiveness.
2) Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down NFTA for the set $M(t)$ of terms which have a ground instance of the term $t = f(a, g(x))$ as a subterm, ie. $M(t) = \Big\{ C\big[f\big(a, g(u)\big)\big] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F}) \Big\}$.
3) Show that NFTAs and top-down DFTAs do not have the same expressiveness.

Exercise 4 : On a more abstract language.

1) Let $\mathcal{E}$ be a finite set of linear terms on $T(\mathcal{F}, \mathcal{X})$. Prove that $Red(\mathcal{E}) = \{C[t\sigma] \mid C \in \mathcal{C}(\mathcal{F}), t \in \mathcal{E}, \sigma \text{ ground substitution}\}$ is recognizable.
2) Prove that if $\mathcal{E}$ contains only ground terms, then one can construct a DFTA recognizing $Red(\mathcal{E})$ whose number of states is at most $n + 2$, where n is the number of nodes of $\mathcal{E}$.

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Homework for next week : Satisfiability

Let $\mathcal{F} = \{and(2), or(2), not(1), 0(0), 1(0), x(0)\}$. A ground term over $\mathcal{F}$ can then be viewed as a boolean formula over x.

1) Give an NFTA which recognizes the set of satisfiable boolean formulae over x.

Now, let $\mathcal{F} = \{and(2), or(2), not(1), 0(0), 1(0), x_1(0), \ldots, x_n(0)\}$, i.e we now handle n variables instead of a single one. The same variable may appear several times in a formula, and should be evaluated consistently.

2) Give an NFTA which recognizes the set of satisfiable boolean formulae over $x_1, \ldots, x_n$.

Note : You can send the homework by mail to asuresh@lsv.fr, or hand it to me in person next time that we reconvene for the TD.